Analysis of a model with a common source of CP violation

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Abstract

We work in a model where all CP violating phenomena have a common source. CP is spontaneously broken at a large scale V through the phase of a complex singlet scalar. An additional $SU(2)_L$ singlet vector-like down-type quark relates this high scale CP violation to low energy. We quantitatively analyze this model in the quark sector. We obtain the numerical values of the parameters of the Lagrangian in the quark sector for a specific ansatz of the 4×4 down-type quark mass matrix where the weak phase is generated minimally. $Z\bar{b}b$ vertex will modify in presence of the extra vector-like down-type quark. From the experimental lower bound of the partial decay width $Z \to \bar{b}b$ we find out the lower bound of the additional down-type quark mass. Tree level flavor changing neutral current appears in this model due to the presence of the extra vector-like down-type quark. We give the range of values of the mass splitting Δm_{B_q} in $B_q^0 - \bar{B}_q^0$ system using SM box, Z mediating tree level and Z mediating one loop diagrams together for both $q = d, s$. We find out the analytical expression for Γ_{12}^q in this model from standard box, Z and Higgs mediated penguin diagrams for $B_q^0 - \bar{B}_q^0$ system, $q = d, s$. From this we numerically evaluate the decay width difference $\left|\Delta\Gamma_{B_q}/\Gamma_{B_q}\right|$. We also find out the numerical values of the CP asymmetry parameters a_J and a_π for the decays $B_d^0 \to J/\psi K_s$ and $B_d^0 \to \pi^+\pi^-$ respectively. We get the lower bound of the scale V through the upper bound of the strong CP phase.

1 Introduction

CP violation is an important phenomena in the context of particle physics and cosmology. CP violation is directly observed in the decays of **K** and **B** mesons. The present experimental results [1] are consistent with the standard model(SM). The single phase present in the Cabibbo,

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Kobayashi, Maskawa(CKM) mixing matrix is responsible for this kind of CP violating phenomena. Present observational data of neutrino oscillations prove that neutrinos are massive. Minimal extension of the SM with neutrino mass generate neutrino mixing similar to the quark mixing. So the phases in the neutrino mixing matrix generate CP violating phenomena in the leptonic sector. The strong CP violation comes from non-perturbative instanton effects in the SM. This leads to the so called strong CP problem for which various solutions have been proposed. The bound on the electric dipole moment of neutron gives the bound on strong CP phase. In the context of cosmology baryon asymmetry in the Universe(BAU) gives an observational evidence for CP violation. Decay of heavy Majorana neutrino to lepton(both charged and neutral) and scalar(both charged and neutral) generate lepton asymmetry which violates CP [2]. One way of generation of baryon asymmetry is through sphaleron mediated process from lepton asymmetry.

Different CP violating processes are not related in general. So the question may arise whether it is possible to find a model where all kinds of CP violation have a common origin. In fact there are a few models [3, 4]. We work in the model proposed by Branco, Parada and Rebelo(BPR) [3]. CP breaks in this model spontaneously through a $SU(3)_C \times SU(2)_L \times U(1)_Y$ singlet complex scalar which gets a vacuum expectation value with nonzero phase, $\frac{V}{\sqrt{2}} \exp(i\alpha)$. This large scale phase is responsible for all kinds of CP violation. A vector-like down-type quark and three generations of right handed neutrinos relate the large scale phase to low energy CP violating phase in the quark sector and the leptonic sector respectively. We quantitatively analyze this model for the quark sector. In Section 2 we describe the model [3] in detail. In Section 3 we calculate parameters of the Lagrangian for a specific ansatz for the down-type quark mass matrix, under the assumption that additional down-type quark mass is larger than the standard down-type quark masses and using the experimental range of values of the CKM parameters and the standard down-type quark masses as the inputs. We take CKM matrix in "standard" parametrization [5] for convenience. Presence of a vector-like down-type quark generate flavor changing neutral current(FCNC) at the tree level. It changes the flavor preserving vertex as well.

We see the effect in $Z \to \bar{b}b$ decay in Section 3. In this Section we get the lower bound of the additional down-type quark mass from the experimental lower bound of $Z \to \bar{b}b$ decay width. We also see the effect in Δm_{B_q} for $q = d, s$ of this additional down-type quark mass in the same Section 3. We also discuss the effect of this model on $\Delta\Gamma_{B_q}$ in the same section for both $q = d, s$. We evaluate the expression of linearly U_{bd} dependent contribution to Γ_{12}^q which is require to calculate the decay width differences. U_{bd} is tree level coupling of b and d quarks with Z and also with neutral physical as well as fictitious scalar. We only consider the light Higgs contribution and disregard heavy scalar contribution. At last in this section we calculate the numerical values of the CP asymmetry parameters a_J and a_π in the decays $B_d^0 \to J/\psi K_s$ and $B_d^0 \to \pi^+\pi^-$ respectively. Using the parametric solution and the extra down-type quark mass bound to strong CP phase we get the lower bound on the scale V in Section 4. Section 5 contains concluding remarks.

2 The Model

The particle content of the standard model is

$$
\begin{aligned}
&{q'}_L = \begin{pmatrix} u'_L \\ d'_L \end{pmatrix} (3, 2, \frac{1}{6}), \quad u'_R(3, 1, \frac{2}{3}), \quad d'_R(3, 1, -\frac{1}{3}), \\
&{\psi^{l'}}_L = \begin{pmatrix} \nu'_L \\ l'_L \end{pmatrix} (1, 2, -\frac{1}{2}), \quad l'_R(1, 1, -1), \quad \Phi = \begin{pmatrix} \phi^+ \\ \frac{\phi_1 + i\phi_2}{\sqrt{2}} \end{pmatrix} (1, 2, \frac{1}{2}).
\end{aligned} \tag{1}
$$

q'_L, ${\psi^{l'}}_L$ and Φ are the standard quark, lepton and scalar doublets respectively. Here u'_L, u'_R, d'_L and d'_R are standard three generation left handed up-type, right handed up-type, left handed down-type and right handed down-type quark fields respectively. l'_L, l'_R and ν'_L are standard three generation left handed lepton, right handed lepton and left handed neutrino fields respectively.

The model of BPR [3] has the same gauge symmetry as the SM, viz., $SU(3)_C \times SU(2)_L \times U(1)_Y$. But in addition it has discrete symmetries $Z_4 \times CP$. There are also six additional multiplets in this model along with the SM particles. One is a totally gauge singlet complex scalar S. Others are the two chiralities of an $SU(2)_L$ singlet vector-like down-type quark D', and three generation of gauge singlet right handed neutrinos N'_R. These additional multiplets have the following $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge group representations

$$
D'_R(3, 1, -\frac{1}{3}), \quad D'_L(3, 1, -\frac{1}{3}), \quad N'_R(1, 1, 0), \quad S(1, 1, 0). \tag{2}
$$

The primes on the fermion fields in Eq. (1) and Eq. (2) imply that they do not necessarily have definite mass. Under Z_4 the fields transform as,

$$
S \to -S, \quad D' \to -D', \quad {\psi^{l'}}_L \to i{\psi^{l'}}_L, \quad l'_R \to il'_R, \quad N'_R \to iN'_R \tag{3}
$$

while all the other fields remain invariant. The Yukawa interactions in this model are,

$$
-\mathscr{L}_q^Y \;=\; \bar{q}'_L Y^d \Phi d'_R + \bar{q}'_L Y^u \widetilde{\Phi} u'_R + (f_q S + {f_q}' S^*) \bar{D}'_L d'_R + \mu_0 \bar{D}'_L D'_R + h.c. \tag{4}
$$

in the quark sector and

$$
-\mathscr{L}_l^Y \;=\; \bar{\psi}^{l'}_L Y^l \Phi l'_R + \bar{\psi}^{l'}_L Y^\nu \widetilde{\Phi} N'_R + {N'}^T_R C^{-1} (f_\nu S + {f_\nu}' S^*) N'_R + h.c. \tag{5}
$$

in the leptonic sector, where $\widetilde{\Phi} = i\sigma_2 \Phi^*$ and S^* is the conjugate field of S. Y^u, Y^d in Eq. (4) and Y^l in Eq. (5) are the standard 3×3 Yukawa coupling matrices. f_q, f'_q in Eq. (4) are the nonstandard 1×3 Yukawa coupling matrices. Y^ν, f_ν and f'_ν in Eq. (5) are also nonstandard 3×3 Yukawa coupling matrices. Without loss of generality we choose the basis of the fields such that Y^u is diagonal with the real positive entities. C in Eq. (5) is a 4×4 matrix which relate any charge conjugated four spinor field to its original field, $\psi^c = \gamma_0 C \psi^*$. The Lagrangian $\mathscr{L}_q^Y$ in Eq. (4) has bare mass term of D' where μ_0 is the bare mass parameter. Presence of this

term does not violate any symmetry of the Lagrangian. All the couplings of the Lagrangian of this model are real due to CP symmetry. The most general gauge, CP and Z_4 invariant scalar potential [6] of this model is

$$\begin{aligned}\mathscr{V} &= -\mu^2\Phi^\dagger\Phi + \lambda(\Phi^\dagger\Phi)^2 + SS^*(a_1 + b_1 SS^*) + (S^2 + S^{*2})(a_2 + b_2 SS^*)\\ &\quad + b_3(S^4 + S^{*4}) + (\Phi^\dagger\Phi)\{c_1(S^2 + S^{*2}) + c_2 SS^*\}.\end{aligned} \tag{6}$$

Because of the terms a_2, b_2, b_3 and c_1 the field S can acquire a complex vacuum expectation value (VEV) : $\langle S\rangle = \frac{V}{\sqrt{2}}\exp(i\alpha)$. This VEV breaks CP and Z_4 spontaneously[1]. The $SU(2)_L \times U(1)_Y$ symmetry spontaneously breaks when the scalar Φ takes VEV, $\langle\Phi\rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}$. It is natural to think that $V \gg v$ due to the fact that the scalar S is a gauge singlet.

3 Quantitative analysis in the hadronic sector

The VEV of Φ and S break the $SU(3)_C \times SU(2)_L \times U(1)_Y \times Z_4 \times CP$ symmetry down to $SU(3)_C \times U(1)_Q \times Z_2$. This generates mass terms. Mass part of the Lagrangian in the quark sector is,

$$-\mathscr{L}_q^m = \bar{d}'_L m_d^0 d'_R + \bar{u}'_L m_u^0 u'_R + M_D^0 \bar{D}'_L d'_R + \mu_0 \bar{D}'_L D'_R + h.c. \tag{7}$$

where,

$$m_u^0 = \frac{v}{\sqrt{2}} Y^u, \quad m_d^0 = \frac{v}{\sqrt{2}} Y^d \tag{8}$$

and

$$M_D^0 = \frac{V}{\sqrt{2}}\{f_q \exp(i\alpha) + f_q{}' \exp(-i\alpha)\}. \tag{9}$$

Since we are in a basis where Y^u is diagonal with the real positive entities, the up-type quark mass matrix m_u^0 will also be diagonal, $m_u^0 = \mathrm{diag}(m_u,\, m_c,\, m_t)$. So the up-type fields u' are physical. We will call them u(unprimed) for future reference. This is our convention. The down-type quark mass term from Eq. (7) can be written by the following form

$$-\mathscr{L}_d^m = \begin{pmatrix} \bar{d}'_L & \bar{D}'_L \end{pmatrix} \begin{pmatrix} \{m_d^0\}_{\mathbf{3\times3}} & \{0\}_{\mathbf{3\times1}} \\ \{M_D^0\}_{\mathbf{1\times3}} & \{\mu_0\}_{\mathbf{1\times1}} \end{pmatrix} \begin{pmatrix} d'_R \\ D'_R \end{pmatrix}. \tag{10}$$

The above mentioned 4×4 down-type quark mass matrix is denoted by $\mathscr{M}$. $\mathscr{M}$ can be diagonalized by bi-unitary transformation,

$$\mathscr{U}^\dagger \mathscr{M} \mathscr{U}' = \begin{pmatrix} \bar{m}_d & 0 \\ 0 & M_D \end{pmatrix}, \tag{11}$$

[1]Spontaneous breaking of any discrete symmetry generates cosmological domain wall problem. Solution of this problem goes beyond the scope of this article.

where $\bar{m}_d = \mathrm{diag}(m_d,\ m_s,\ m_b)$ and M_D is the mass of additional down-type quark. Let us consider the forms of $\mathscr{U}$ and $\mathscr{U}'$ as follows

$$\mathscr{U} = \begin{pmatrix} K_{\mathbf{3}\times\mathbf{3}} & R_{\mathbf{3}\times\mathbf{1}} \\ S_{\mathbf{1}\times\mathbf{3}} & T_{\mathbf{1}\times\mathbf{1}} \end{pmatrix}, \quad \mathscr{U}' = \begin{pmatrix} K'_{\mathbf{3}\times\mathbf{3}} & R'_{\mathbf{3}\times\mathbf{1}} \\ S'_{\mathbf{1}\times\mathbf{3}} & T'_{\mathbf{1}\times\mathbf{1}} \end{pmatrix}. \tag{12}$$

Relations of the physical basis(unprimed) of the down-type quark fields to the original basis are

$$\begin{pmatrix} d_L \\ D_L \end{pmatrix} = \begin{pmatrix} K^\dagger & S^\dagger \\ R^\dagger & T^* \end{pmatrix} \begin{pmatrix} d'_L \\ D'_L \end{pmatrix} \tag{13}$$

and

$$\begin{pmatrix} d_R \\ D_R \end{pmatrix} = \begin{pmatrix} K'^\dagger & S'^\dagger \\ R'^\dagger & T'^* \end{pmatrix} \begin{pmatrix} d'_R \\ D'_R \end{pmatrix}. \tag{14}$$

Clearly $\mathscr{U}$ diagonalizes $\mathscr{M}\mathscr{M}^\dagger$:

$$\mathscr{U}^\dagger \mathscr{M}\mathscr{M}^\dagger \mathscr{U} = \begin{pmatrix} \bar{m}_d^2 & 0 \\ 0 & M_D^2 \end{pmatrix}. \tag{15}$$

This gives the approximate relation [3, 6]

$$\mathscr{H} = K \bar{m}_d^2 K^{-1} \tag{16}$$

where

$$\mathscr{H} = m_d^0 m_d^{0\dagger} - \frac{m_d^0 M_D^{0\,\dagger} M_D^0 m_d^{0\dagger}}{M_D^2} \tag{17}$$

is the effective mass matrix square for the standard down-type quarks under the assumption

$$\frac{\bar{m}_d^2}{M_D^2} \ll 1, \tag{18}$$

and with

$$M_D^2 \approx M_D^0 M_D^{0\,\dagger} + \mu_0^2. \tag{19}$$

K is the CKM matrix and it is approximately unitary for the hermiticity of $\mathscr{H}$. The other blocks of $\mathscr{U}$ in Eq. (12), under the assumption in Eq. (18) obtain the forms [6]

$$R \approx \frac{m_d^0 M_D^{0\,\dagger}}{M_D^2}, \qquad T \approx 1 \tag{20}$$

and

$$S \approx -\frac{M_D^0 m_d^{0\dagger} K}{M_D^2}. \tag{21}$$

The constraints of unitarity of the matrix $\mathscr{U}$ defined in Eq. (12), imply the relations

$$\sum_j |K_{ij}|^2 = 1 - |R_i|^2 \tag{22}$$

for all generations i, $i = u, c, t$. This implies that the sum of all couplings of any of the up-type quarks to all the standard down type quarks is not same for all generations. Presence of $|R_i|^2$ term in the right hand side of Eq. (22) breaks the weak universality. Smallness of the 2nd term in the right hand side of Eq. (22) due to the assumption in Eq. (18) suggests that unitarity of CKM matrix K and weak universality hold only approximately in this model.

Here the CKM phase can be generated through the second term of $\mathscr{H}$ in Eq. (17). Note that this term is not suppressed by the scale V because M_D^0 and M_D, as we have seen from Eq. (9) and Eq. (19), both have dependence on the scale V.

Now we try to analyze this model quantitatively. Since up-type quark mass matrix is diagonal, the remaining parts of $\mathscr{L}_q^m$ in Eq. (7) have 18 real parameters. These parameters are 9 of m_d^0, 3 of f_q, 3 of f_q' and other 3 are α, V and μ_0. The matrix $\mathscr{M}$ in Eq. (11) and hence $\mathscr{H}$ in Eq. (17) contains all these parameters. The question may arise whether it is possible to find realistic solution of the Eq. (16) for the entire range of values of the CKM parameters and the masses of the standard down-type quarks. Since $\mathscr{H}$ is hermitian, there are 9 independent coupled equations consisting of these 18 parameters as variables. The reduction of the number of variables can be obtained as the following way. First we assume that m_d^0 is symmetric. Then we note that M_D and M_D^0 occur in Eq. (17) in the combination

$$F_D = \frac{M_D^0}{M_D}, \tag{23}$$

which can be parametrized as follows

$$F_D^T = \begin{pmatrix} f_1 \exp(i\alpha) + f_1' \exp(-i\alpha) \\ f_2 \exp(i\alpha) + f_2' \exp(-i\alpha) \\ f_3 \exp(i\alpha) + f_3' \exp(-i\alpha) \end{pmatrix}, \tag{24}$$

where $f_i = \frac{V}{\sqrt{2}M_D}(f_q)_i$ and $f_i' = \frac{V}{\sqrt{2}M_D}(f_q')_i$. Now if we consider $\mathbf{f} = \mathbf{f}'$, the matrix $\mathscr{H}$ in Eq. (17) becomes real symmetric. This implies that the CKM phase is zero. Now let us see minimally how we can generate a non zero CKM phase. To this end, we take the ansatz

$$\begin{aligned} f_1 &= f_1' = f \\ f_2 &= f_2' = \beta f \\ f_3 &= f + \Delta f \\ f_3' &= f. \end{aligned} \tag{25}$$

So ultimately we have 10 variables which are f, Δf, α, β and six elements of m_d^0. So we are in a position where we can find the parameters of the Lagrangian from Eq. (16). We choose the values of the CKM parameters and the masses of the standard down-type quarks from their

Type of Input Parameters	Specification	Experimental Range	Random Choice
CKM Parameters	$\sin\theta_{12}$	0.2227 to 0.2259	0.2238
	$\sin\theta_{13}$	0.0032 to 0.0042	0.0037
	$\sin\theta_{23}$	0.0398 to 0.0428	0.0398
	δ_{CKM}	46° to 74°	73.6818°
Mass Parameters	m_d	4.0 to 8.0 MeV	5.9171 MeV
	m_s	80.0 to 130.0 MeV	81.6310 MeV
	m_b	4.1 to 4.4 GeV	4.3358 GeV

Table 1: Here θ_{ij}'s are the CKM angles and δ_{CKM} is CKM phase in "standard" parametrization of CKM matrix [5]. Experimental range of the sin of the CKM angles, CKM phase and the standard down-type quark masses are taken from PDG [1]. In [1] the values quoted for mass of m_d, m_s are measured at the scale 2 GeV and $m_b \equiv \bar{m}_b(\bar{m}_b)$ in the $\overline{\mathrm{MS}}$ scheme. We randomly choose a set of the above inputs from their experimental range.

experimental range as in Table 1 . Using these values in the right hand side of the Eq. (16) we solve this equation for different values of β. We observe that there is a range of the values of β, $-3.2 < \beta < 2.12$, where Eq. (16) fails to give real solutions for the parameters of $\mathscr{H}$ for the entire range of values of the CKM parameters and the standard down-type quarks masses as in Table 1. We get the real solutions outside this range of values of β.

To get the essence of the solutions let us show that how we have proceeded to reach the goal. First we write m_d^0 and F_D in the following forms:

$$m_d^0 = v \begin{pmatrix} x_1 & x_2 & x_3 \\ x_2 & x_4 & x_5 \\ x_3 & x_5 & x_6 \end{pmatrix} \qquad {F_D}^T = \sqrt{x_7} \begin{pmatrix} 1 \\ \beta \\ x_8 + i x_9 \end{pmatrix} . \tag{26}$$

Here x_1 to x_6 are six independent dimensionless mass parameters of m_d^0. x_7 to x_9 are three independent dimensionless parameters of F_D. We have written F_D in Eq. (26) with a pre-factor $\sqrt{x_7}$. This makes x_7 dependence of Eq. (16) linear. The x_7 to x_9 are related to f, Δf and α in the following way

$$x_7 = 4 f^2 \cos^2\alpha, \quad x_8 = 1 + \frac{\Delta f}{2f}, \quad x_9 = \frac{\Delta f \tan\alpha}{2f}. \tag{27}$$

For every set of inputs we have four sets of solutions. These can be divided into two pairs of sets, where within a pair, the two solutions are related only by change of signs for x_1 to x_6. The solutions of different pairs have the values of the same order of magnitudes. We provide two distinct sets of solutions in Table 2 for a set of inputs in the rightmost column of Table 1 with $\beta = 3.5$ and $\beta = -3.5$. Inputs are chosen randomly from their experimental range in Table 1.

Now let us see that what are the outcomes of these solutions. m_d^0 becomes explicitly known. F_D also becomes known meaning that Yukawa parameters related to the coupling of the standard down-type quarks, extra vector like down-type quark and the singlet scalar also become

Variables	Results For β=3.5		Results For β=−3.5	
	Set No. 1	Set No. 2	Set No. 1	Set No. 2
x_1	0.000162841	0.000168356	−0.000008467	−0.0000154413
x_2	0.000301087	0.000320389	0.000298491	0.000322901
x_3	−0.000255079	0.000229067	−0.000320499	0.000291728
x_4	−0.000122682	−0.0000551233	0.000131765	0.0000463298
x_5	−0.000845869	0.000848642	0.00107484	−0.00106796
x_6	−0.0212162	0.0212857	0.026958	−0.0267854
x_7	0.0488582	0.0539958	0.0289893	0.0353266
x_8	−2.53447	2.14896	4.41581	−3.72177
x_9	−0.849496	0.766168	−1.28386	1.06035

Table 2: Two distinct sets of solutions of the Eq. (16) for the inputs of the rightmost column of Table 1 with $\beta = \pm 3.5$.

Quantities	Results For β=3.5		Results For β=−3.5	
	Set No. 1	Set No. 2	Set No. 1	Set No. 2
f	±0.113667	±0.139648	±0.0909458	±0.0963174
Δf	∓0.803504	±0.3209	±0.621307	∓0.909577
α	13.5145	33.6967	−20.5991	−12.6567
$F_D F_D^\dagger$	0.996472	0.996495	0.997163	0.997127

Table 3: Values of f, Δf, α and $F_D F_D^\dagger$ for the solution in Table 2 using Eq. (27).

known upto a factor M_D/V. Using the relations in Eq. (27) we obtain the values of f, Δf and α in Table 3 for the different solutions in Table 2. The R block of $\mathscr{U}$ as in Eq. (12) is the coupling of charged current interactions among up-type quarks u and D. $S^\dagger S$ is a matrix which generates FCNC for the standard down-type quarks. Using Eq. (23), we can rewrite Eq. (20) and Eq. (21) as

$$\begin{aligned} R &= \frac{m_d^0 {F_D}^\dagger}{M_D} \\ S &= -\frac{F_D {m_d^0}^\dagger K}{M_D}. \end{aligned} \tag{28}$$

The numerator of R and S in above Eq. (28) become known from the numerical solution. Their forms for the Set-1 solution with $\beta = 3.5$ in Table 2 are

$$R = \frac{1}{M_D}\begin{pmatrix} 0.101309 - i0.0117825 \\ 0.109596 - i0.0390722 \\ 2.74902 - i0.980013 \end{pmatrix} \quad S^T = \frac{1}{M_D}\begin{pmatrix} -0.0993492 - i0.000938182 \\ -0.0228038 - i0.000215232 \\ -2.75133 - i0.980436 \end{pmatrix}. \tag{29}$$

Now let us observe two parts of the $\mathscr{H}$ matrix in Eq. (17). Using the Set-1 solution for $\beta = 3.5$

in Table 2 we have

$$m_d^0 m_d^{0\dagger} = \begin{pmatrix} 0.0110282 & 0.0137888 & 0.309574 \\ 0.0137888 & 0.0496956 & 1.08766 \\ 0.309574 & 1.08766 & 27.287 \end{pmatrix} \quad (30)$$

and

$$\frac{m_d^0 M_D^{0\dagger} M_D^0 m_d^{0\dagger}}{M_D^2} =$$
$$\begin{pmatrix} 0.0104023 & 0.0115634 + i0.00266705 & 0.290048 + i0.0668936 \\ 0.0115634 - i0.00266705 & 0.0135379 & 0.339572 - i5.14869 \times 10^{-6} \\ 0.290048 - i0.0668936 & 0.339572 + i5.14869 \times 10^{-6} & 8.51754 \end{pmatrix}. \quad (31)$$

It is interesting to see that two parts of $\mathscr{H}$ contributes roughly equally for many elements. The second part of $\mathscr{H}$ in Eq. (31) is new physics term and it is not suppressed. This feature is independent of the inputs we are giving. Another input independent feature is that the contribution of bare part to the mass of D quark M_D in Eq. (19) is smaller than the Yukawa part. The Yukawa part $M_D^0 M_D^{0\dagger}/M_D^2 = F_D F_D^\dagger$ contributes approximately 99.64% to M_D^2 as in Table 3 for the solutions in Table 2 with $\beta = 3.5$. The contribution for the solutions $\beta = -3.5$ is nearly 99.71% as in Table 3. Although bare mass term is gauge invariant, its contribution to the mass of D quark is subdominant. One conclusion we may draw from here that M_D should be less than V for the validity of perturbation theory. Now let us see that how we can get more information about M_D from $Z \to b\bar{b}$ decay and how M_D value affects $B_d^0 - \bar{B}_d^0$ mixing.

The presence of extra vector quark generates FCNC at the tree level. It also changes the flavor preserving neutral current. The interaction of Z and the standard down-type quark is

$$\mathscr{L}_{NC}^d = -\frac{g}{2\cos\theta_W} \sum_{i,j=1}^{3} \bar{d}_i \gamma_\mu (g_V - \gamma_5 g_A)_{ij} d_j Z^\mu \quad (32)$$

where $d_1 = d$, $d_2 = s$, $d_3 = b$,

$$(g_V)_{ij} = g_V^{SM} \delta_{ij} - I_3^d U_{ij} \quad (33)$$

and

$$(g_A)_{ij} = g_A^{SM} \delta_{ij} - I_3^d U_{ij} \quad (34)$$

where

$$U = S^\dagger S \quad (35)$$

and the tree level couplings in SM are

$$\begin{aligned} g_V^{SM} &= I_3^d - 2Q \sin^2\theta_W, \\ g_A^{SM} &= I_3^d \end{aligned} \quad (36)$$

with $I_3^d = -1/2$, $Q = -1/3$. The flavor preserving vertex appreciably changes by loop corrections. We want to see the effect of loop correction along with the new contribution of U_{bb} dependent terms in $Z \to b\bar{b}$ process [7]. The partial decay width of $Z \to b\bar{b}$ in terms of effective coupling constants $g^b_{V_{\rm eff}}$ and $g^b_{A_{\rm eff}}$ is [8]

$$\begin{aligned}\Gamma_{Z\to b\bar{b}} = \frac{\sqrt{2}G_F M_Z^3}{4\pi} \quad & [(1-4y)^{\frac{1}{2}}\{|g^b_{V_{\rm eff}}|^2(1+2y) + |g^b_{A_{\rm eff}}|^2(1-4y)\} \times (1+\delta_{QED}) \\ & +(|g^b_{V_{\rm eff}}|^2 + |g^b_{A_{\rm eff}}|^2)\delta_{QCD}] + \Delta_{QCD}(y) \end{aligned} \tag{37}$$

where $y = m_b^2/M_Z^2$, QED correction δ_{QED} and factorizable QCD correction δ_{QCD} are

$$\begin{aligned}\delta_{QED} &= 3\alpha Q^2/4\pi, \\ \delta_{QCD} &= (\alpha_s/\pi) + 1.41(\alpha_s/\pi)^2 - 12.8(\alpha_s/\pi)^3 - \alpha\alpha_s Q^2/4\pi^2, \end{aligned} \tag{38}$$

and the effective couplings are

$$\begin{aligned} g^b_{V_{\rm eff}} &= \sqrt{\rho_b}(I_3^b - I_3^b U_{bb} - 2Q\sin^2\theta_W \kappa_b) \\ g^b_{A_{\rm eff}} &= \sqrt{\rho_b}(I_3^b - I_3^b U_{bb}). \end{aligned} \tag{39}$$

Using general expression for ρ_b and κ_b [8, 9, 10] with three loop QCD correction and two loop electroweak correction, we obtain $0.9935 \le \rho_b \le 0.9941$ and $1.0341 \le \kappa_b \le 1.0382$ for SM using experimental range of values of $\alpha_s(M_Z)$, m_t, M_W, M_Z and m_b [1]. Hence, we get that the SM prediction is $0.37655 \le \Gamma_{Z\to b\bar{b}} \le 0.37869$ GeV. $\Delta_{QCD}(y)$ in Eq. (37) is b quark mass dependent QCD correction. We disregard here small non-factorizable QCD correction. New physics in the form of U_{bb} can decrease $g^b_{V_{\rm eff}}$, $g^b_{A_{\rm eff}}$ and hence the decay width $\Gamma_{Z\to b\bar{b}}$, since by definition $0 \le U_{bb} \le 1$. Experimental data at the 1σ level given $0.37593 \le \Gamma_{Z\to b\bar{b}} \le 0.37912$ GeV [1]. If we take the SM contribution at the theoretical lower limit 0.37655 GeV, we would then need

$$U_{bb} \le 7.204 \times 10^{-4} \tag{40}$$

in order that the new physics contributions do not violate the experimental bound. From the definition of matrix U in Eq. (35) and using the form of S in Eq. (28) we see that U is proportional to $1/M_D^2$ which is unknown. Hence we can use the upper bound on U_{bb} to obtain a lower bound on M_D. We provide these bounds in Table 4 for different values of β. The upper bound of the value of the other elements of U become known from the lower bound on M_D. For the different sets of solution M_D values are approximately same for a given set of inputs. The least value of D quark mass $M_D \approx 40$ GeV for the given range of β in the Table 4. The maximum value among the elements in matrix $\bar{m}_d$ is b quark mass which is nearly 4 GeV. So the assumption $\bar{m}_d^2/M_D^2 \ll 1$ in Eq. (18) is not too bad at all. This assumption is better for smaller β region where M_D has to be larger.

We now want to see the effect in flavor changing process. U dependent extra piece in g_V and g_A in Eq. (33) and Eq. (34) are responsible for the FCNC. These generate tree level Z mediating $B_q^0 - \bar{B}_q^0$ mixing, where $q = d, s$. The experimental value of the mass splitting Δm_{B_d} in $B_d^0 - \bar{B}_d^0$ system is well known, $\Delta m_{B_d}^{\rm ex} = (3.304 \pm .046) \times 10^{-10}$ MeV [1]. For $B_s^0 - \bar{B}_s^0$ system

only lower bound exists, $\Delta m_{B_s} > 94.8\times10^{-10}$ MeV at 95% CL [1]. The contribution of SM box [11], Z mediated tree diagrams [12, 13] and Z mediated one loop diagrams together in Δm_{B_q} has been explictly calculated by Barenboim and Botella [14][2], who give off diagonal term of $B_q^0-\bar{B}_q^0$ mixing matrix M_{12}^q as

$$M_{12}^q = M_{12}^{qSM}\Delta_{bq}^* \tag{41}$$

where

$$M_{12}^{qSM} = \frac{G_F^2 M_W^2 \eta_B m_{B_q}(B_{B_q} f_{B_q}^2)}{12\pi^2}\bar{E}(x_t)(\xi_t^{q*})^2, \tag{42}$$

and

$$\Delta_{bq} = 1 - a(U_{bq}/\xi_t^q) - b(U_{bq}/\xi_t^q)^2. \tag{43}$$

Here a and b are

$$\begin{aligned} a &= 4\frac{\bar{C}(x_t)}{\bar{E}(x_t)} \\ b &= \frac{4\pi\sin^2\theta_W}{\alpha}\frac{1}{\bar{E}(x_t)} \end{aligned} \tag{44}$$

where

$$\begin{aligned} \bar{E}(x_t) &= \frac{-4x_t+11x_t^2-x_t^3}{4(1-x_t)^2} + \frac{3x_t^3\ln x_t}{2(1-x_t)^3} \\ \bar{C}(x_t) &= \frac{x_t}{4}\left[\frac{4-x_t}{1-x_t} + \frac{3x_t\ln x_t}{(1-x_t)^2}\right]. \end{aligned} \tag{45}$$

The parameters involved in M_{12}^{qSM} in Eq. (42) m_{B_q} and f_{B_q} are mass and decay constant of B_q^0 meson respectively. B_{B_q} is Renormalization Group invariant parameter. η_B is QCD factor. Its value is nearly 0.55 [15]. $m_{B_d} = 5279.4\pm0.5$ MeV and $m_{B_s} = 5369.6\pm2.4$ MeV [1]. $x_t = m_t^2/M_W^2$. $\xi_t^q = V_{tb}^* V_{tq}$, $q=d,s$. The uncertainty in the calculation come from $\sqrt{B_{B_q}}f_{B_q}$. $\sqrt{B_{B_d}}f_{B_d} = 221\pm28^{+0}_{-22}$ MeV and $\sqrt{B_{B_s}}f_{B_s} = 255\pm31$ MeV which are obtained from the lattice QCD calculations in [16]. Mass splitting Δm_{B_q} is defined in terms of M_{12}^q

$$\Delta m_{B_q} = 2\,|M_{12}^q|\,. \tag{46}$$

The range of SM predictions for the range of values of m_t, M_W, m_{B_q} and $\sqrt{B_{B_q}}f_{B_q}$ are $\Delta m_{B_d}^{SM} = (2.292 \text{ to } 5.318)\times10^{-10}$ MeV and $\Delta m_{B_s}^{SM} = (81.991 \text{ to } 146.373)\times10^{-10}$ MeV. Here CKM matrix element V_{td}, V_{ts} and V_{tb} are same as K_{31}, K_{32} and K_{33} respectively. We use $V_{td} = 0.0079 - i0.00347$, $V_{ts} = -0.03906 - i0.000796$ and $V_{tb} = 0.9992$ which are obtained using randomly chosen CKM parameters in the rightmost column in Table 1. The range of Δm_{B_s}

[2] Although the authors in [14] calculated Δm_B for $B_d^0-\bar{B}_d^0$ system, it can be easily extended to $B_s^0-\bar{B}_s^0$ system also.

and Δm_{B_d} for the different values of β are shown in Table 4 and 5 respectively. We see that the range has a small negative shift in the negative β region and a small positive shift in the positive β region from SM range for $B_d^0 - \bar{B}_d^0$ system. The feature is reversed for $B_s^0 - \bar{B}_s^0$ system where range of Δm_{B_s} has a small positive shift in the negative β region and a small negative shift in the positive β region from SM range.

Now we want to see the new physics effect on the decay width difference $\Delta\Gamma_{B_q}$ in $B_q^0 - \bar{B}_q^0$ system for both $q = d, s$. The SM results come only from box diagrams [11]. Presence of tree level FCNC coupled to Z and physical Higgs generate new contributions to the decay width difference in the $B_q^0 - \bar{B}_q^0$ system through penguin diagrams. We disregard Higgs mediated contributions to M_{12}^q which are subdominant compared to the Z mediated contributions. We now want to calculate the absorptive part of the amplitude Γ_{12}^q for $B_q^0 \to \bar{B}_q^0$ in this model, keeping only terms linear in U_{bq}. Standard box, Z and Higgs mediated one loop penguin diagrams give

$$\Gamma_{12}^q = \frac{G_F^2 m_b^2 m_{B_q}(B_{B_q} f_{B_q}^2)}{8\pi}\left[f_{SM}^q(z_c) + U_{bq}\sum_{i=2}^{3}\xi_i^q\left\{2(f_{\rm box}(i,1) - f_{\rm box}(1,1)) + f_{\rm pen}(i) - f_{\rm pen}(1)\right\}\right] \tag{47}$$

where SM result due to box diagrams come through $f_{SM}^q(z_c)$ [11, 17] whose expression is

$$f_{SM}^q(z_c) = \xi_t^{q2} + \frac{8}{3}\xi_t^q\xi_c^q(z_c + \frac{1}{4}z_c^2 - \frac{1}{2}z_c^3) + \xi_c^{q2}\left\{\sqrt{1-4z_c}(1 - \frac{2}{3}z_c) + \frac{8}{3}z_c + \frac{2}{3}z_c^2 - \frac{4}{3}z_c^3 - 1)\right\}. \tag{48}$$

The absorptive function [11] related to SM box is

$$\begin{aligned} f_{\rm box}(i,j) &= \frac{1}{3(1-x_i)(1-x_j)}\sqrt{1+(z_i-z_j)^2 - 2(z_i+z_j)}\times \\ &\left[\left\{(1+\frac{x_i x_j}{4})(3-(z_i+z_j)-(z_i-z_j)^2)\right\} + 2x_b(z_i+z_j)(z_i+z_j-1)\right] \end{aligned} \tag{49}$$

whereas the absorptive function related to penguin diagrams mediated by Z and light Higgs is

$$\begin{aligned} f_{\rm pen}(i) = 4\times\frac{-4x_Z\sqrt{1-4z_i}}{3(1-x_i)(x_b-x_Z)}&\left[\left\{z_i(g_R^u - g_L^u)(1-\frac{x_b}{4}-\frac{x_i}{2}) + \frac{g_L^u}{3}(1+2z_i-\frac{3}{2}x_i) + \frac{g_R^u x_i}{6}(1+2z_i)\right\}\right. \\ &\left.-\frac{5}{8}\left\{\frac{g_L^u}{3}(1+2z_i) + \frac{x_i g_R^u}{3}(1-z_i) + \frac{x_i}{2x_Z}(1-x_i) + \frac{x_b x_i}{8x_Z}(1+2z_i)(\frac{x_Z-x_H}{x_b-x_H}) + \frac{3x_i^2}{4x_Z}(\frac{x_b-x_Z}{x_b-x_H})\right\}\right]. \end{aligned} \tag{50}$$

Here $\xi_i^q = V_{ib}^* V_{iq}$, $z_i = m_i^2/m_b^2$, $x_i = m_i^2/M_W^2$, $x_b = m_b^2/M_W^2$, $x_Z = M_Z^2/M_W^2$, $g_L^u = 1/2 - 2/3\sin^2\theta_W$ and $g_R^u = -2/3\sin^2\theta_W$. $i, j = (1, 2, 3 \equiv u, c, t)$. The expression $f_{\rm box}(i,j)$ and $f_{\rm pen}(i)$ are only for $i, j = 1, 2$. Presence of top inside the loop does not contribute to the absorptive part due to the kinematical impossibility. For penguin diagrams with single up-type quarks inside the loop does not contribute to the absorptive part. Here the function in Eq. (50) is for

two up-type quarks inside the loop. The 4 factor present infront of the expression of $f_{\rm pen}(i)$ in Eq. (50) is due to the fact that there are four types of diagrams contributing equally. These are S and T channel diagrams with loop at the different vertices. Now the definition of $\Delta\Gamma_{B_q}$ [17] is

$$\Delta\Gamma_{B_q} = \frac{4{\rm Re}(M_{12}\Gamma_{12}^{q*})}{\Delta m_{B_q}}. \tag{51}$$

We numerically evaluate the range of $\left|\Delta\Gamma^{B_q}/\Gamma_{B_q}\right|$ using range of quark masses, B_q^0 masses, decay constants and life times of B_q^0. $m_c = 1.15$ to 1.35 GeV[1] where $m_c \equiv \bar{m}_c(\bar{m}_c)$. $\tau_{B_d} = 1/\Gamma_{B_d} = (1.536 \pm 0.014) \times 10^{-12}$s and $\tau_{B_s} = 1/\Gamma_{B_s} = (1.461 \pm 0.057) \times 10^{-12}$s [1]. Experimental value of the other parameters are given previously. The SM range of $\left|\Delta\Gamma^{B_d}/\Gamma_{B_d}\right| = (0.3845$ to $0.9066)\%$ and $\left|\Delta\Gamma^{B_s}/\Gamma_{B_s}\right| = (12.5191$ to $24.0086)\%$. The exprimental constraints are $\left|\Delta\Gamma^{B_d}/\Gamma_{B_d}\right| < 18\%$ at 90% CL [1, 18] and $\left|\Delta\Gamma^{B_s}/\Gamma_{B_s}\right| < 54\%$ at 95% CL [1, 19]. In our analysis we disregard the masses of d, s and u quarks. We also disregard the contributions from heavy scalar meidiated penguins to Γ_{12}^q in Eq. (47). Their contributions are suppressed by their mass. We have shown our results of $\left|\Delta\Gamma^{B_d}/\Gamma_{B_d}\right|$ in Table 5 and $\left|\Delta\Gamma^{B_s}/\Gamma_{B_s}\right|$ in Table 4 for the different values of the parameter β. We have seen that there is small positive shift of decay width in the negative β region and small negative shift in the positive β region for the $B_d^0 - \bar{B}_d^0$ system. On the contrary for $B_s^0 - \bar{B}_s^0$ system we have seen that there is small negative shift of decay width in the negative β region and small positive shift in the positive β region.

Single vector like down type quark model has simple expression for CP asymmetry of B_d^0 decay to two channels $J/\psi K_s$ and $\pi^+\pi^-$. Those are [14]

$$\begin{aligned} a_J &\equiv \frac{\Gamma(B^0 \to J/\psi K_s) - \Gamma(\bar{B}^0 \to J/\psi K_s)}{\sin(\Delta m_{B_d} t)(\Gamma(B^0 \to J/\psi K_s) + \Gamma(\bar{B}^0 \to J/\psi K_s))} \\ &= \sin(2\beta_0 - \arg(\Delta_{bd})) \end{aligned} \tag{52}$$

and

$$\begin{aligned} a_\pi &\equiv \frac{\Gamma(B^0 \to \pi^+\pi^-) - \Gamma(\bar{B}^0 \to \pi^+\pi^-)}{\sin(\Delta m_{B_d} t)(\Gamma(B^0 \to \pi^+\pi^-) + \Gamma(\bar{B}^0 \to \pi^+\pi^-))} \\ &= \sin(2\alpha_0 + \arg(\Delta_{bd})) \end{aligned} \tag{53}$$

where α_0 and β_0 are defined as usual

$$\alpha_0 = \arg(-\frac{\xi_t^d}{\xi_u^d}) \tag{54}$$

$$\beta_0 = \arg(-\frac{\xi_c^d}{\xi_t^d}). \tag{55}$$

We have calculated the numerical values of those CP asymmetry parameters a_J and a_π in this model. The SM value of those parameters are $a_J = 0.7370$ and $a_\pi = 0.2549$ using CKM

elements with randomly chosen CKM parameters in Table 1. The quantity Δ_{bd} is defined in Eq. (43). We disregard the variaton of $\arg(\Delta_{bd})$ with respect to the change of parameters inside Δ_{bd} for a particular β. This variation is smaller compared to the variation with respect to β. So, inspite of getting range we have a value for those CP asymmetry parameters for a particular β value. We show the results in Table 5. We have seen that value of the asymmetry parameters have been changed in the third place and somewhere in the second place after decimal for different values of β from the SM values.

Here we cannot have any information about the scale V because of the cancellations of the scale from the numerator and the denominator in the second part of $\mathscr{H}$ as Yukawa part dominates over bare part in the mass of D. So the solutions are scale independent. To know about the scale we look for use of these solutions in strong CP.

4 Constraints from the Strong CP Phase

The existence of topologically nontrivial gauge transformations, and of field configurations which make transitions between the different topological sectors of the theory, leads to the existence of the new term $\Theta\frac{g_s^2}{32\pi^2}F^a_{\mu\nu}\tilde{F}^{a\mu\nu}$ in QCD action. This term violates parity P and time-reversal T. It will violate CP due to CPT invariance. Diagonalization of the mass matrix requires different $U(1)$ rotation of the left handed fields and the right handed fields. So it effectively generates nonzero chiral rotation. This modifies the Θ to $\bar{\Theta}$, where $\bar{\Theta} = \Theta + \arg(\det \mathbf{m})$, where $\mathbf{m}$ is quark mass matrix which appear in $\bar{\Psi}_L \mathbf{m} \Psi_R$, where Ψ contains all quarks. Now the model in which we are working, CP is invariant in the Lagrangian and breaks only spontaneously. So we cannot keep the Θ term in our Lagrangian. The mass terms are generated through spontaneous breaking of $SU(2)_L \times U(1)_Y$ and Z_4 symmetry. Since Θ is zero, we can write $\bar{\Theta} = \arg(\det \mathbf{m})$. At the tree level $\det \mathbf{m}$ is real and hence $\bar{\Theta}$ is zero. We should look for one loop correction of the quark mass matrix. Now, loop correction to $\bar{\Theta}$ is

$$\begin{aligned} \delta\bar{\Theta} &= \arg\{\det\,(\mathbf{m} - \Sigma)\} \\ &= -\mathbf{Im}\{\mathbf{Tr}(\mathbf{m}^{-1}\Sigma)\}, \end{aligned} \tag{56}$$

where Σ is self energy matrix. The one loop correction to the mass matrix has been explicitly done by Weinberg in [20]. Goffin, Segre and Weldon [21] have shown that only loops containing physical scalars give nonzero contribution to the strong CP phase. Yukawa coupling and mass matrix of the up-type quarks are real. So they cannot contribute to strong CP phase. Due to this fact the strong CP phase becomes

$$\delta\bar{\Theta} = -\mathbf{Im}\{\mathbf{Tr}(\mathscr{M}^{-1}\Sigma)\}. \tag{57}$$

where $\mathscr{M}$ is the 4×4 down-type quark mass matrix as in Eq. (10) and Σ is now the 4×4 down-type quark self energy matrix. In this model there are three scalar h, s, and t which are originated from fluctuation about the vacuum, $\phi_1 \to v + h$ and $S \to \frac{1}{\sqrt{2}}(V + s + it)\exp(i\alpha)$.

These fields are not mass eigenstates. The physical scalars H_k are related to these scalars $h_a(h,s,t)$ by orthogonal transformations, $H_k = R_{ka}h_a$. In this basis the Yukawa couplings Γ_a of the Lagrangian change to Γ_k, where $\Gamma_k = R_{ka}\Gamma_a$. The contribution of the physical scalars to the self energy matrix is [22]

$$\Sigma^{\phi} = -\sum_k \frac{1}{(4\pi)^2}\int_0^1 dx[(1-x)\mathscr{M}\Gamma_k^{\dagger} + \Gamma_k\mathscr{M}^{\dagger}]\ln\{\mathscr{M}\mathscr{M}^{\dagger}x^2 + M_k^2(1-x)\}\Gamma_k. \tag{58}$$

So the strong CP phase will be

$$\delta\bar{\Theta} = \sum_k \frac{1}{(4\pi)^2}\int_0^1 dx\mathbf{Im}[\mathbf{Tr}\{\mathscr{M}^{-1}\Gamma_k\mathscr{M}^{\dagger}\ln(\mathscr{M}\mathscr{M}^{\dagger}x^2 + M_k^2(1-x))\Gamma_k\}]. \tag{59}$$

We can write the strong CP phase in terms of original Yukawa couplings of the Lagrangian. Changing also the logarithmic part of Eq. (59) in the diagonal basis the dominant part of the strong CP phase comes from D quark mass. Then the strong CP phase will be

$$\delta\bar{\Theta} = \sum_{k,a,b} \frac{R_{ka}R_{kb}}{(4\pi)^2}\int_0^1 dx\mathbf{Im}[\mathscr{U}^{\dagger}\Gamma_a\mathscr{M}^{-1}\Gamma_b\mathscr{M}^{\dagger}\mathscr{U}]_{44}\ln({M_D}^2x^2 + M_k^2(1-x)). \tag{60}$$

The above expression gives a nonzero value only when $a = t$ and $b = h$. The explicit calculations of strong CP phase have been done earlier [6]. To keep the hierarchy between the scales v and the scale V and to avoid the fine tuning in the stationarity equations of the scalar potential in Eq. (6) we should consider two parameters c_1 and c_2 of the scalar potential small as $O(v^2/V^2)$. Light scalar mass M_1 remains of the order of v. The other two scalar masses become $M_2 \sim M_3 \sim V$. So the calculation of the order of magnitude of $R_{kt}R_{kh} \sim 2\sin(2\alpha)\frac{v^3}{V^3}$[6]. In performing integrations we keep the assumption $M_{2,3} \geq 2M_D$ same as in [6] but $2M_D \geq M_1$ is not valid for all β as in Table 4 and as we consider $M_1 = M_H = 150$ GeV. Under the considerations M_2 and M_3 are nearly same and $M_{2,3} \gg 2M_D$ we get the following expression of strong CP phase

$$\delta\bar{\Theta} \approx \frac{\sin 2\alpha}{16\pi^2}(\mathbf{f}_q^2 - \mathbf{f'}_q^2)\frac{v^2}{V^2}\left[1 + \ln\left(\frac{V^2}{M_D^2}\right) - I_1\right] \tag{61}$$

where

$$I_1 = \begin{cases} \frac{M_1^2}{M_D^2}\left[\frac{1}{2}\ln\left(\frac{M_1^2}{M_D^2}\right) + \left(\frac{4M_D^2}{M_1^2} - 1\right)^{\frac{1}{2}}\tan^{-1}\left(\frac{4M_D^2}{M_1^2} - 1\right)^{\frac{1}{2}}\right] & \text{for } 2M_D \geq M_1, \\ \frac{M_1^2}{M_D^2}\left[\frac{1}{2}\ln\left(\frac{M_1^2}{M_D^2}\right) - \left(1 - \frac{4M_D^2}{M_1^2}\right)^{\frac{1}{2}}\tanh^{-1}\left(1 - \frac{4M_D^2}{M_1^2}\right)^{\frac{1}{2}}\right] & \text{for } 2M_D \leq M_1. \end{cases} \tag{62}$$

The integrals are performed by Bento, Branco and Parada in [6]. Using our ansatz of the mass matrix of down-type quarks we have strong CP phase of the following form

$$\delta\bar{\Theta} \approx \frac{\sin 2\alpha}{8\pi^2}\Delta f(\Delta f + 2f)\frac{M_D^2v^2}{V^4}\left[1 + \ln\left(\frac{V^2}{M_D^2}\right) - I_1\right]. \tag{63}$$

β	Lower Bound of M_D[GeV]	Upper bound of $\lvert U_{bs}\rvert \times 10^5$	Range of $\Delta m_{B_s} \times 10^{10}$[MeV]	Range of $\left\lvert \frac{\Delta\Gamma_{B_s}}{\Gamma_{B_s}} \right\rvert \times 10^2$	Lower Bound of V [TeV]
-10.0	59.1	0.838	82.038 to 146.459	12.5139 to 23.9984	10.1
-7.5	72.9	0.695	82.033 to 146.449	12.5145 to 23.9995	12.9
-5.0	108.1	0.506	82.023 to 146.432	12.5155 to 24.0016	17.4
-4.5	123.4	0.461	82.021 to 146.427	12.5158 to 24.0022	19.1
-4.0	146.6	0.414	82.018 to 146.422	12.5161 to 24.0028	21.6
-3.5	187.5	0.364	82.015 to 146.417	12.5165 to 24.0034	25.3
-3.2	234.2	0.333	82.013 to 146.413	12.5167 to 24.0038	29.0
2.2	272.8	0.379	81.965 to 146.328	12.5219 to 24.0140	30.5
2.5	191.3	0.421	81.963 to 146.324	12.5222 to 24.0146	24.7
3.0	136.2	0.493	81.958 to 146.316	12.5226 to 24.0155	19.9
3.5	108.8	0.563	81.955 to 146.309	12.5231 to 24.0163	17.1
4.0	91.9	0.631	81.951 to 146.302	12.5235 to 24.0171	15.1
4.5	80.2	0.697	81.947 to 146.297	12.5238 to 24.0178	13.7
5.0	71.6	0.761	81.945 to 146.292	12.5241 to 24.0184	12.6
7.5	49.7	1.034	81.937 to 146.278	12.5249 to 24.0200	9.3
10.0	41.2	1.220	81.938 to 146.280	12.5248 to 24.0197	6.9

Table 4: The lower bound on M_D using the obtained upper bound of $U_{bb} \leq 7.204 \times 10^{-4}$ for the different values of β. The upper bound of U_{bs}, range of Δm_{B_s}, range of $\left\lvert \frac{\Delta\Gamma_{B_s}}{\Gamma_{B_s}} \right\rvert \times 10^2$ and the lower bound of the scale V for the different values of β. We use the solutions for the inputs which are in the rightmost column of Table 1. We use $\alpha(M_Z) = 1/128.91$, $\alpha_s(M_Z) = (0.1187 \pm 0.002)$, $M_Z = (91.187 \pm 0.0021)$ GeV, $M_W = (80.425 \pm 0.038)$ GeV, $M_H = 150$ GeV, $\sin^2\theta_W = 1 - M_W^2/M_Z^2$, $m_t = (174.3 \pm 5.1)$ GeV and $m_b = 4.1$ to 4.4 GeV [1].

The strong phase will vanish for Δf=0, α=0, $\pi/2$, π, 2π. Weak phase will also vanish for those values of Δf and α because for each value $M_D^{0\,\dagger} M_D^0$ and hence $\mathscr{H}$ become real symmetric. In terms of x parameters the form of strong CP phase using the relations in Eq. (27) we have

$$\delta\bar{\Theta} \approx \frac{1}{4\pi^2} x_7 x_8 x_9 \frac{M_D^2 v^2}{V^4}\left[1 + \ln\left(\frac{V^2}{M_D^2}\right) - I_1\right]. \tag{64}$$

Present value of the experimental bound on electric dipole moment of neutron gives value of upper bound on strong CP phase, $\delta\bar{\Theta} \leq 2 \times 10^{-10}$. We can convert the bound of $\delta\bar{\Theta}$ to bound of the scale V using the Eq. (64), the solutions of the Eq. (16) and the bound of M_D in Table 4. We show the lower bounds on the scale V for the different values of β in Table 4. The bounds on the scale V for the different sets of solution for the given inputs will differ within 1 TeV. There exists a tiny window on the scale V for the bound of $\delta\bar{\Theta}$ for the expression in Eq. (64) which is small like v. We discard this result because the result come for fine cancellation among the terms inside the parenthesis in Eq. (64).

β	Upper bound of $\lvert U_{bd}\rvert \times 10^5$	Range of $\Delta m_{B_d} \times 10^{10}$[MeV]	Range of $\left\lvert\frac{\Delta\Gamma_{B_d}}{\Gamma_{B_d}}\right\rvert \times 10^2$	a_J for $B_d \to J/\psi K_s$	a_π for $B_d \to \pi^+\pi^-$
-10.0	3.652	2.273 to 5.274	0.3864 to 0.9108	0.7434	0.2641
-7.5	3.027	2.275 to 5.279	0.3863 to 0.9108	0.7418	0.2615
-5.0	2.204	2.278 to 5.287	0.3861 to 0.9102	0.7403	0.2597
-4.5	2.009	2.279 to 5.289	0.3859 to 0.9100	0.7400	0.2592
-4.0	1.802	2.280 to 5.291	0.3859 to 0.9097	0.7396	0.2587
-3.5	1.585	2.282 to 5.294	0.3858 to 0.9095	0.7392	0.2581
-3.2	1.449	2.282 to 5.295	0.3857 to 0.9093	0.7390	0.2578
2.2	1.649	2.307 to 5.353	0.3829 to 0.9026	0.7359	0.2534
2.5	1.836	2.309 to 5.358	0.3827 to 0.9022	0.7360	0.2535
3.0	2.146	2.312 to 5.365	0.3824 to 0.9013	0.7361	0.2536
3.5	2.451	2.316 to 5.373	0.3820 to 0.9005	0.7363	0.2540
4.0	2.748	2.319 to 5.381	0.3817 to 0.8997	0.7367	0.2545
4.5	3.037	2.323 to 5.388	0.3814 to 0.8990	0.7371	0.2551
5.0	3.316	2.326 to 5.396	0.3811 to 0.8982	0.7377	0.2559
7.5	4.507	2.338 to 5.423	0.3798 to 0.8952	0.7416	0.2615
10.0	5.317	2.341 to 5.431	0.3792 to 0.8936	0.7458	0.2676

Table 5: The upper bound of U_{bd}, range of Δm_{B_d}, range of $\left\lvert\frac{\Delta\Gamma_{B_d}}{\Gamma_{B_d}}\right\rvert \times 10^2$, CP asymmetry parameters a_J and a_π for the decay channels $B_d^0 \to J/\psi K_s$ and $B_d^0 \to \pi^+\pi^-$ respectively for the different values of β. We use the solutions for the inputs which are in the rightmost column of Table 1. M_D values are taken from the Table 4 to calculate U_{bd} bound. We use also the inputs in the caption of Table 4. [1].

5 Conclusion

The model of BPR [3] claims that all CP violating phenomena can originate from a single phase which appears in the VEV of SM gauge singlet complex scalar S. Bento, Branco and Parada [6] started this work where they showed that the weak CP phase and the strong CP phase can have a common origin. BPR [3] extend this to the leptonic sector.

We have made quantitative studies of this model in the quark sector. We find the Lagrangian parameters for a specific kind of ansatz of down-type quark mass matrix where CKM phase is generated minimally. The Lagrangian of the BPR model is CP invariant. CP is broken spontaneously through a single phase α in the VEV of the singlet scalar S. Hence, all Lagrangian parameters are real and the phases are only due to α. We observe that in the interval $-3.2 < \beta < 2.12$ the ansatz of Eq. (25) becomes inconsistent. We find the numerical value of the Lagrangian parameters outside this interval. We also find that the contribution of the bare part to the mass of D quark is negligible compare to the Yukawa part. This observation is

independent of β. One point we should make about the unitarity of $\mathscr{U}$ Eq. (12). To calculate U we use the solution of the Eq. (16) where K is unitary. We calculate approximate R, S and T of Eq. (20) and Eq. (21) with the solutions Set-1 for $\beta = 3.5$ in Table 2 and the value of M_D for the same solution in Table 4. Putting these R, S and T and unitary K in $\mathscr{U}$ Eq. (12) to test its unitarity we see that $\mathscr{U}\mathscr{U}^\dagger$ and $\mathscr{U}^\dagger\mathscr{U}$ deviate from identity by the amount only $O(10^{-4})$ for a few elements. This deviation is not so sensitive to the parameter β.

We have observed that this kind of vector quark model cannot increase the partial decay width of $Z \to \bar{b}b$. So we find out the bound on U_{bb} from the condition that the new physics decrease the SM value up to experimental lower bound of $\Gamma_{Z\to\bar{b}b}$ considering 1σ error of the total decay width of Z and the branching ratio of this decay mode of Z. Here we also should point out about value of M_H. Suppose we fix the value of β at 3.5 and change M_H from 150 GeV to 100 GeV. Then the decay width of $Z \to \bar{b}b$ decreases by nearly 0.01 MeV, M_D remains almost same whereas V increases by nearly 0.34 TeV. On the other hand, for the same value of β, if M_H is changed from 150 GeV to 300 GeV, it increases the decay width of $Z \to \bar{b}b$ by nearly 0.007 MeV, M_D remains almost same whereas V decreases by nearly 0.62 TeV. Scope of new physics remains in determining the mass difference Δm_{B_q} for both $q = d, s$. Here we have only small shift of the Δm_{B_q} range from SM range. Actually huge uncertainty of $\sqrt{B_{B_d}}f_{B_d} = 221 \pm 28^{+0}_{-22}$ and $\sqrt{B_{B_s}}f_{B_s} = 255 \pm 31$ introduce huge uncertainty in theoretical value of Δm_{B_d} and Δm_{B_s} respectively. This model has very small effect on decay width difference in both $B^0_d - \bar{B}^0_d$ and $B^0_s - \bar{B}^0_s$ system. The results are shown in Table 5 and 4 respectively. We also have observed in Table 5 that the numerical values of the CP asymmetry parameters a_J and a_π for the decays $B^0_d \to J/\psi K_s$ and $B^0_d \to \pi^+\pi^-$ respectively have effect on the third and somewhere on the second place after the decimal from SM value. This part of our analysis is quite general and applies to any model containing extra down-type quarks, e.g., models inspired by E_6 grand unification.

The strong CP phase in this model is suppressed by inverse powers of V. It should be noted that we have used the solutions of the elements of the quark mass matrix to obtain lower bounds on V. There are, however, direct experimental limits on the mass of strongly produced massive quarks of charge $-1/3$. This lower bound of 199 GeV [23] is derived for a fourth generation down-type quark which is produced strongly in pair and decay to bZ via 1-loop FCNC. If this bound is assumed to hold for M_D in the BPR model where FCNC exists at the tree level, the lower bound on V increases. For example, with $\beta = -10$, we now get $V > 20.3$ TeV, whereas for $\beta = +10$, we get $V > 15.4$ TeV.

Note added: After completion of our work a new analysis of a group was published [24], where the experimental value of m_b has changed to a larger value with a smaller error bar, $m_b = 4.591 \pm 0.040$ GeV in the kinetic scheme at scale 1 GeV. But we have used the PDG [1] value $m_b = 4.1$ to 4.4 in our numerical analysis in this model, and the random value chosen in Table 1 for finding Lagrangian parameters lies outside the 1σ range advocated in [24].

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